SOME MATHEMATICS OF THE CALCULATING MACHINE

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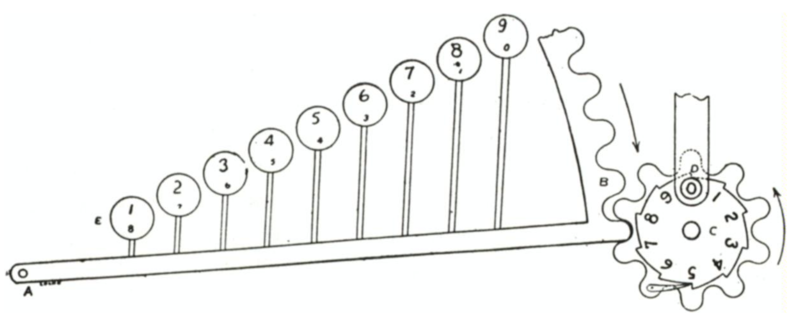
There are certain relatively simple properties of numbers which would be deemed of little importance in the science of numbers except for their application in the field of machine calculation. In considering such properties no detailed knowledge of the mechanical features of the machine is necessary. We are concerned only with the *set-up* and final *recording* or *registering dials*. One unit of our primitive machine will consist of a set of nine keys numbered from 1 to 9, and a registering dial actuated by the keys in such a manner as to turn one tenth of a revolution for each unit in the number on the key pressed down. In the diagram, if O appears at window D, over registering dial C, and if key 7 is pushed down, the segmental gear B causes dial C to rotate in a counter clockwise direction until 7 appears at the window. If key 5 is then operated the dial C is turned 5 more spaces and 2 appears at the window. This primary actuation will be called *digital addition*, as only the unit's digit of the sum is registered.



If, as the 9 leaves the window, means is provided to move the dial in the next, or ten's place, forward one step, i. e., to provide for the *carry*, the primitive machine is complete. Neglecting the mechanical features necessary to make it operable, we have essentially an *adding* machine. It is the purpose to set forth what may be done with such a machine and why.

Multiplication is performed by repeating the addition. To multiply 849 by 37, use the 9 key in the first column, 4 key in the ten's column or second column, and the 8 key in the third or hundred's column. Obviously the operation of this set of keys simultaneously 37 times would accomplish the multiplication. The work in such a case would be prohibitive. After the set has been operated 7 times each finger is shifted one place to the left, placing the finger on the 9 in ten's column, etc., and operated 3 times, i. e., multiplying by 3 and by 10 at the same time.

Subtraction is an inverse process. To make the machine process an exact parallel of the arithmetical, the registering dials must rotate in the opposite direction when subtracting. With an adding machine, as described above, the registering dials rotate in but one direction. Subtraction on such a machine is performed by what may be called *over-addition,* in the explanation of which process several terms will be needed.

The *complement of a number* is the difference between that number and the next higher power of ten. The complement of 6 is 4.

The *co-digit* of a digit is the difference between that digit and 9. The co-digit of 6 is 3.

The *supplement*[1] *of a number* will indicate the difference between that number and an infinitely great power of ten. The supplement of 257 is . . . 999999643.

To write the supplement of a number, write the complement of the right hand significant figure, proceed to the left with the co-digits in order and complete the capacity of the machine with 9s. Some machines are equipped with a device to cut out the carry mechanism at any desired point. This device renders the prefixed 9s unnecessary.

Examples

1.	2.	3.	4.	5.
7351	7351	7351	7351	7351
—1867	+8133	+99998133	—1800	+8200
5484	1/5484	00005484	5551	1/5551

[1] The writer suggests this word to signify a concept for which no term has been adopted. The 'infinite' phase of the definition amounts in practice to prefixing sufficient 9's to prolong the carry beyond the capacity of the machine.

Examples 1 and 4 indicate arithmetical subtraction. In 4, the 8 is the right hand significant figure, the complement of which is 2 in example 5.

Examples 2, 3 and 5 indicate machine subtraction. In 3 the supplement of the subtrahend is used; in 2 and 5 the carry cut-off is indicated by the line, the 1 at its left not appearing on the machine.

To simplify the picking out of the co-digits in subtraction, the co-digit appears in small or in red type on the digit key. Since the complement of the right hand significant figure of the subtrahend is used and not the co-digit, the operator must move the finger up one key for this particular order.

The algebraic equivalence of the results is seen in the case of a three order number as follows:

Subtraction

$$\begin{array}{lll} 100c & +10b & +a \\ 100z & +10y & +x \\ \hline 100(c-z) & +10(b-y) & +(a-x) \end{array}$$

Adding the Supplement to the Subtrahend

$$\begin{array}{lll} 100c & +10b & +a \\ +100(9-z) & +10(9-y) & +(10-x) \\ \hline \end{array}$$

$$900 + 100(c-z) + 90 + 10(b-y) + 10 + (a-x) =$$
$$1000 + /100(c-z) + 10(b-y) + (a-x)$$

Division. Division may be performed on an adding machine by repeating the subtraction, shifting the subtrahend to the right when the partial dividend becomes smaller than the divisor. The subtraction is performed by overaddition and the quotient figures are written as found. The process may be described as follows:

Set up the dividend.

Depress the keys indicating the supplement of the number as far to the left as possible, using the co-digit keys as in subtraction. Repeat the depression of this set of keys, counting the depressions, until the partial dividend is less than the divisor. Shift the fingers each one place to the right and continue.

It will be evident that such a process requires either a very intimate knowledge of the science of numbers or sufficient train-

ing to make the process mechanical, to give the operator that sense of confidence in the work which is essential.

Division on an adding machine is simplified somewhat by using the complement of the subtrahend rather than the supplement, by virtue of the following unique property of numbers. It will be noted that while the use of the supplement cuts out the carry mechanism immediately at the left of the number, the use of the complement does not.

If the complement of the divisor is repeatedly added to the partial dividend until the number of times the addition takes place is equal to the digit on the left of the partial dividend, including the accretions from the carrys, the partial dividend at that point is the actual dividend remaining in that division which produces a quotient figure equal to the number of additions at this point. An example will make this clear. Let it be required to divide 258 by 37. 63 is the complement of 37.

Machine Work

	No. of additions
2 | 58	
63	
3 | 21	1
63	
3 | 84	2
63	
4 | 47	3
63	
5 | 10	4
63	
5 | 73	5

By Division

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37)258(5
   185
   ---
    73
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In this example, when 5 additions have been made, the right hand figure of the sum is 5. At this point the right hand part of the sum is the partial dividend 73 when the quotient figure is 5. If this partial dividend is still greater than the divisor, continue the process in this order. In the example given, the 73 is the new true partial dividend and the process of division begins anew at this point, giving, in this case, one more unit in the present order of the quotient.

The significance of the above property of numbers will be lost unless it be kept in mind that every division is made up of a series of division examples, each example having its own partial dividend formed when the next figure is brought down and the division beginning anew at this point. With the machine process each new division begins at that point when the true partial dividend appears on the machine. A little reflection will show that the separation into simple divisions in the machine process does not always coincide with that of the arithmetical process.

The following algebraic treatment of the property is readily seen to be general.

In dividing $100a + 10b + c$ by $10d + e$ let $a < d$ and let the quotient be $a + n$ and the remainder be $10g + h$.

$$10d + e \;)\; 100a + 10b + c \;(\; a + n$$

$$10g + h \text{ remainder}$$

The a is the first digit in the dividend and the n is the accretions from the carrys. To prove that the addition of the complement of the divisor $(a + n)$ times produces the correct partial dividend corresponding to the subtraction of the divisor $(a + n)$ times.

$$\text{The complement of } 10d + e \text{ is } 100 - (10d + e \text{ or}$$

$$10(9 - d) + 10 - e$$

$$(a + n)\ [100 - (10d + e)] =$$

$$100\ (a + n) + 100a + 10b + c - 10ad - 10nd - ae - ne$$

It is necessary to show that this expression lacking the first member is equal to the remainder $10g + h$.

$$(10d + e)\ (a + n) + 10g + h = 100a + 10b + c$$

$$\text{or, } 10g + h = 100a + 10b + c - 10ad - 10nd - ae - ne$$

As each step of the division is performed the quotient figure appears in its proper place following the first one, until the division is completed. The final appearance of the machine is quotient | remainder, the $a + n$ being the quotient.

If the complement of the divisor is made up of many orders or if the proper keys are too inconveniently placed to be easy of manipulation, the above process is apt to be unwieldy. Short division by this process is tedious and for it is substituted multiplication by the reciprocal of the divisor, a table of reciprocals being provided. Square root by subtraction of successive odd numbers and the principles underlying automatic division will be considered in another article.